Kirchhoff-Born migration

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Summary

The classical Kirchhoff migration method assumes an asymptotic ray theory form of the Green's function. To be consistent with ray theory assumptions, the ray slowness model that produces this function must be a smooth version of the true slowness model. However, the implied wavefield extrapolation through the smooth model introduces diffraction errors. These diffraction errors are predicted via a first order Born approximation that includes a surface and volume integral term. The surface integral represents the wavefield extrapolation through the smooth slowness model and the volume integral is the slowness perturbation contribution to the wavefield propagator.

Introduction

Kirchhoff migration methods are the most computationally efficient procedures for imaging 3-D wavefield data. For general slowness models, the migration wavefield propagation operators, Green's functions, are usually obtained by ray methods. For example, Gray and May, 1994, discuss a finite difference eikonal approach. However certain imaging errors occur due to single ray path assumptions (see Fei, et al, 1995). These errors include incomplete focusing (first arrival time events do not represent the dominant amplitude events) and erroneous focusing (difficulty of predicting ray amplitudes at caustics).

Ruhl (1996) presented a derivation of the one-way wave equation starting from a wavenumber domain formulation of the Kirchhoff-Born equation. Relationships to the split-step method were also shown. This implies the Kirchhoff-Born equation models high order wave equation phenomena.

In this paper, an integral formulation of the Kirchhoff-Born equation is presented. This equation predicts a diffraction term that is missing in the ray-based Kirchhoff imaging procedure that is significant for large lateral slowness perturbations. We derive this diffraction term in the context of a Kirchhoff migration integral and propose a method for calculating this term.

Born and Kirchhoff integral

The constant density acoustic wave equation is

$$\nabla^2 P[x,\omega] + \omega^2 s^2 P[x,\omega] = 0 \qquad [1]$$

where P is the pressure wavefield, ω is the angular frequency, s is the slowness, and x is the spatial location of the wavefield. To obtain the Born approximation, we introduce the slowness perturbation, $\Delta s[x]$,

$$s[x] = s_o[x] + \Delta s[x] \qquad [2]$$

about the reference slowness, $s_o[x]$, which is assumed to be a sufficiently smoothly varying slowness function that it can be used to calculate ray travel times.
Equation [1] can be written as

$$\nabla^2 P[x,\omega] + \omega^2 s_o^2 P[x,\omega] = -2\omega^2 s_o \Delta s\, [1 + \Delta s/ 2s_o]\, P[x,\omega] \qquad [3]$$

Using the Green's function, G_o, for the ray reference slowness model, the solution to equation [3] in integral form, is

$$P[x',\omega] = \int d\Gamma\, n \bullet \{ \nabla G_o[x,x']\, P[x,\omega] - G_o[x,x']\, \nabla P[x,\omega] \} + 2\omega^2 \int dx''\, G_o[x'',x']\, e[x'']\, P[x'',\omega] \qquad [4]$$

where $P[x',\omega]$ is the interior pressure wavefield, Γ is the bounding surface with x' and x'' as interior points, n is the surface normal vector, G_o satisfies

$$\nabla^2 G_o[x,x'] + \omega^2 s_o^2 G_o[x,x'] = \delta[x - x'] \qquad [5]$$

and the Green's function is calculated using asymptotic ray theory to solve the eikonal and transport equations, i.e.

$$G_o[x,x'] \approx A[x,x'] \exp[\, i\omega\, t[x,x']\,]$$

where t[x,x'] satisfies the eikonal equation

$$\nabla t \bullet \nabla t = s_o^2$$

and the amplitude term, A[x,x'], satisfies the transport equation

$$2 \nabla t \bullet \nabla A + A \nabla^2 t = 0 .$$

e[x] is the virtual volume source term

$$e[x] = s_o \Delta s [1 + \Delta s / 2s_o] . \qquad [6]$$

Note here that G_o does not designate the analytic Green's function for the constant slowness case, but for a Green's function determined from a slowness model that is consistent with ray theory.

The surface integral term in equation [4] represents the contribution to the wavefield at x' via extrapolation through the reference slowness model. To compute the ray path using ray theory, the Courant-Friedrichs-Lewy condition requires that $|\nabla s/s|\ \delta\lambda < 1$, where $\delta\lambda$ is the incremental ray path length. The correction term is the volume integral which represents a superposition of secondary sources that are excited by the incident wavefield P when it encounters the slowness perturbations e[x], The secondary sources are propagated by G_o to x'. The first order Born approximation used to derive [3] assumes

1. small magnitude perturbations, $|\ \Delta s/s_o\ | < 1$, and
2. the incident wavefield is the extrapolated wavefield given by

$$P[x,\omega] = \int d\Gamma\ n' \bullet \{\nabla G_o[x,x']\ P[x',\omega] - G_o[x,x']\ \nabla P[x',\omega]\}.$$

Assume G_o is the Dirichlet Green's function, G_{do}, that is zero on the boundary; under these assumptions equation [4] is approximated as

$$\begin{aligned} P[x',\omega] \approx & \int d\Gamma\ n \bullet\ \nabla G_{do}[x,x']\ P[x,\omega] + \\ & + 2\omega^2 \int dx''\ G_{do}[x'',x']\ e[x''] \bullet \\ & \bullet \int d\Gamma\ n \bullet\ \nabla G_{do}[x,x'']\ P[x,\omega] \end{aligned}$$

Interchanging the volume and surface integrals, we have

$$\begin{aligned} P[x',\omega] \approx & \int d\Gamma\ P[x,\omega]\ \{\ n \bullet\ \nabla G_{do}[x,x'] + \\ & + 2\omega^2 \int dx''\ n \bullet\ \nabla G_{do}[x,x'']\ e[x'']\ G_{do}[x'',x']\ \}. \qquad [7] \end{aligned}$$

The volume scattering integral is an effective wavefield propagation operator that includes the higher order diffraction terms that were neglected in the high frequency asymptotic calculation of G_{do}.

This term is significant for large slowness perturbations and yields a coda that extends the wavefield propagator in time. To obtain a zero-offset migration expression from equation [7], integrate the interior wavefield, $P[x',\omega]$, with respect to frequency to invoke the imaging condition.

If we assume that the virtual source term e[x] has bounded support, e.g. it is non-zero only at large discontinuities such as the boundary of a salt body , then the S-matrix approach from quantum mechanics (see Merzbacher, 1997) is a computationally attractive method for evaluating the volume integral. The S or scattering matrix gives the amplitude and phase for plane wave to plane wave scattering from a scattering potential e[x'']. The volume integral is

$$V = 2\omega^2 \int dx''\ n \bullet\ \nabla G_{do}[x,x'']\ e[x'']\ G_{do}[x'',x'] \qquad [8]$$

and substituting the ray form of G_{do} and neglecting the gradient of the amplitude which is a near field term, one obtains

$$\begin{aligned} V = & 2i\omega^3\ n \bullet p[x] \int dx''\ A[x,x'']\ A[x'',x'] \bullet \\ & \bullet \exp[\ i\omega\ \{t[x,x''] + t[x'',x']\}\ ]\ e[x''] \qquad [9] \end{aligned}$$

where A[x,x''] is the ray amplitude from x to x'', t[x,x''] is the corresponding ray travel time, and *p*[x] is the ray slowness vector. Assuming e[x''] has bounded support; let q designate a reference point for the source volume. To evaluate the volume integral, equation 9, we use the stationary phase method (see Bleistein and Handelsman, 1975).

$$\begin{aligned} V = & 2i\omega^{3/2}\ n \bullet p[x]\ A[x,q]\ A[q,x'] \cdot \bullet \\ & \bullet \exp[\ i\omega\ \{t[x,q,x'] + i\ \pi/4\ \mathrm{sgn}\{\ t[x,q,x'],xx\ \}\ ] \\ & \bullet E[k_s]\ /\ \ \det[\ t[x,q,x'],xx\ ] \qquad [10] \end{aligned}$$

where t[x,q,x'] = t[x,q] + t[q,x'], $E[k_s]$ is the spatial Fourier transform of e[x''], k_s is the scattering wavenumber vector,

$$k_s = \omega\ p[x,q] + \omega\ p[q,x'],$$

p[x,q] is the entrance slowness vector and the prime designates the exit slowness vector. The expression, t[x,q,x'],xx , is the time wavefront curvature at point q.

By evaluating V and adding it to the integrand of equation [7], we have included the first-order diffraction term to the Kirchhoff migration approach. V can be calculated independently of the surface integral so that one can investigate when and where the diffraction corrections need to be included in the migration.

Conclusions

A Kirchhoff-Born equation was developed to describe surface wavefield extrapolation via asymptotic ray theory Green's functions. It was shown that due to the assumptions required for the slowness model used in ray tracing, a high frequency diffraction term was neglected. This diffraction term comes from a volume scattering that is the difference of the true slowness model and the ray slowness model. Accompanying this derivation of the Kirchhoff-Born equation was a scattering matrix-Fourier transform method to compute this diffraction term.

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